

# Determining the Neutral D Mixing Parameters

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Evidence for mixing in the neutral  $D$  meson system has recently been reported. Assuming negligible CP violation, non-vanishing width and mass differences between the two neutral  $D$  mass eigenstates has been found. Theoretical predictions of these are rather difficult, obscuring detection of New Physics contributions. However, the observation of CP violation in the  $D$  system would be a good signal of New Physics. We briefly describe the formalism that describes the neutral  $D$  decay and mixing, and present a method to determine all the mixing parameters accurately allowing for arbitrary CP violation.

## 1. INTRODUCTION

Neutral meson mixing is a flavor changing process, resulting in a flavor change  $\Delta F = 2$  and within the Standard Model it occurs via box diagrams with internal quarks and  $W$  bosons. While the charged current flavor violating interactions can appear at the tree level, flavor changing neutral current (FCNC) interactions are feasible only at the loop level. Hence these interactions have an important role to play in the search for Physics beyond the Standard Model (SM), or New Physics(NP), as the new particles could appear virtually in the loops. One hopes to constrain NP by a measurement of these FCNC processes.

Until March 2007, neutral meson mixing had only been seen in the down type mesons:  $K$ (in 1956),  $B_d$ (in 1987) and  $B_s$ (in 2006). The parameter  $\epsilon_K$  in  $K^0 - \bar{K}^0$  mixing played a constraining role in building of models of NP. The measured mixing in the  $B_d$  and  $B_s$  mesons, being consistent with SM predictions has also resulted in constraining various NP models.  $D$  meson is the only up type meson where mixing is possible. Evidence for mixing in the neutral  $D$  meson system has recently been reported [1, 2, 3, 4, 5] by the Belle, BaBar and CDF collaborations. These experiments find non-vanishing width and mass differences between the two neutral  $D$  mass eigenstates assuming negligible CP violation (CPV). The HFAG average for ICHEP08 [6] rules out the no mixing scenario at  $9.8\sigma$ . In the Standard model one expects the mixing parameters in the D system to be small. Further, decays of the  $D$  meson via tree diagrams as well as  $D^0 - \bar{D}^0$  mixing- due to the negligible contribution of the internal  $b$  quark in the box diagram, essentially involve only two generations. Hence one expects that there should be no CPV in the charm system.

## 2. THEORETICAL ESTIMATES AND LOOKING FOR NEW PHYSICS

While the internal charm quark makes the dominant contribution in the box diagram for  $K^0 - \bar{K}^0$  mixing, the large top mass is responsible for appreciable mixing in  $B_d$  and  $B_s$  mesons inspite of the suppression due to the small CKM elements. In fact, mixing in the neutral  $K$  and  $B_d$  systems resulted in predictions for the charm and top quark masses respectively, before direct discovery. However, in the  $D$  meson system, the heaviest down type quark, the  $b$  quark is not heavy enough to compensate for the large suppression due to the small CKM factor,  $|V_{ub}V_{cb}^*|^2$ . Contributions from the internal light quarks ( $s$  or  $d$ ) are dominant and hence, the mixing parameters in the D system are expected to be small. In the flavor SU(3) limit one would expect mixing to exactly vanish. In fact, it has been shown to arise only at second order in SU(3) breaking [7].

An explicit calculation of the SM mixing parameters in the  $D$  system is very hard. This is due to the fact that the  $D$  meson mass lies in the intermediate range where neither the inclusive nor the exclusive approaches work too well. An inclusive approach is based on an operator product expansion in terms of matrix elements of local operators of increasing dimensions with coefficients in powers of  $\Lambda/m_c$ . However,  $m_c$  is not large enough to allow the expansion upto few terms to be accurate. Such calculations [10] suggest mass and width differences of order  $10^{-3}$ . On the

other hand, in an exclusive approach, summing over hadronic final states also fails as  $D$  is not light enough, that just a sum over few exclusive channels could suffice. Moreover, cancellations between states within a given SU(3) multiplet requires that the contribution of each state be known with high precision. In Ref. [7] SU(3), breaking arising from phase space differences was studied and it was concluded that the width difference could be at the level of one percent.

Since the theoretical predictions of the  $D$  mixing parameters by various groups vary over orders of magnitude, a clear signal of NP from comparison of the expected and observed values of the mass and width differences may be hard. However, an observation of CPV will imply presence of NP [8], independent of hadronic uncertainties. In most extensions of SM, the decay amplitudes and width difference are not expected to be affected by NP, however, the mass difference could be modified by new short distance CP violating contributions. In our discussion, we consider CPV only in the mixing and no direct violation [9].

### 3. FORMALISM AND NOTATION

Time evolution of the  $D$  system is governed by the Schrodinger equation:

$$i \frac{\partial}{\partial t} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \mathbf{H} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \begin{pmatrix} M_{11} - i \frac{\Gamma_{11}}{2} & M_{12} - i \frac{\Gamma_{12}}{2} \\ M_{12}^* - i \frac{\Gamma_{12}^*}{2} & M_{22} - i \frac{\Gamma_{22}}{2} \end{pmatrix} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} \quad (1)$$

where  $M_{11} = M_{22}$ ,  $\Gamma_{11} = \Gamma_{22}$  from CPT invariance. The neutral  $D$  mass eigenstates are related to the weak eigenstates by,  $|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$ , where  $\frac{q}{p} = \sqrt{\frac{M_{12}^* - i \frac{\Gamma_{12}^*}{2}}{M_{12} - i \frac{\Gamma_{12}}{2}}}$  with  $|p|^2 + |q|^2 = 1$  is obtained by diagonalizing the Hamiltonian matrix. The off diagonal elements of the mass matrix are due to transitions via off-shell intermediate states while those of the decay matrix are from on shell states and therefore constitute respectively the the dispersive and absorptive parts of  $D$  mixing. If the magnitude of  $q/p$  differs from unity and/or the weak phase  $\phi = \arg(q/p)$  is nonvanishing, this would signal  $CP$  violation.

The time evolution of the states  $|D^0(t)\rangle$  and  $|\bar{D}^0(t)\rangle$  which start of as pure  $|D^0\rangle$  and  $|\bar{D}^0\rangle$  at  $t = 0$  is given by

$$|D^0(t)\rangle = f_+(t)|D^0\rangle + \frac{q}{p}f_-(t)|\bar{D}^0\rangle, \quad (2)$$

$$|\bar{D}^0(t)\rangle = \frac{p}{q}f_-(t)|D^0\rangle + f_+(t)|\bar{D}^0\rangle, \quad (3)$$

where,

$$f_+(t) = e^{-iMt - \frac{\Gamma t}{2}} \cos\left(\frac{\Delta M t}{2} - \frac{i\Delta\Gamma t}{4}\right), \quad (4)$$

$$f_-(t) = -e^{-iMt - \frac{\Gamma t}{2}} i \sin\left(\frac{\Delta M t}{2} - \frac{i\Delta\Gamma t}{4}\right), \quad (5)$$

and  $M$  and  $\Gamma$  are the average mass and width of the two mass eigenstates, also the the mass and width differences of these eigenstates are popularly written [11] in terms of the dimensionless variables,

$$x \equiv \frac{\Delta M}{\Gamma} = \frac{M_1 - M_2}{\Gamma} \quad \text{and} \quad y \equiv \frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}.$$

The time evolution functions  $f_{\pm}(t)$  are studied in the limit  $x \ll 1$ ,  $y \ll 1$  and  $t\Gamma \ll 1$ . In this limit we have

$$f_+(t) = e^{-iMt - \frac{\Gamma t}{2}} \left[ 1 - \frac{(x^2 - y^2)}{8} \Gamma^2 t^2 + \mathcal{O}(\Gamma^4 t^4) \right] \quad (6)$$

$$f_-(t) = -i e^{-iMt - \frac{\Gamma t}{2}} \left[ \frac{(x - iy)}{2} \Gamma t + \mathcal{O}(\Gamma^3 t^3) \right] \quad (7)$$

Hence, the time dependent decay rates for a  $D^0$  decaying to a final state  $f$  and  $D^0 \rightarrow f$  and  $\bar{D}^0 \rightarrow \bar{f}$  have the form:

$$|A(D^0(t) \rightarrow f)|^2 = e^{-\Gamma t} [X_f + Y_f \Gamma t + Z_f (\Gamma t)^2 + \dots] \quad (8)$$

$$|A(\bar{D}^0(t) \rightarrow \bar{f})|^2 = e^{-\Gamma t} [\bar{X}_f + \bar{Y}_f \Gamma t + \bar{Z}_f (\Gamma t)^2 + \dots]. \quad (9)$$

#### 4. MEASURING THE MIXING PARAMETERS

This time dependence is utilized in determining the mixing parameters. At  $t = 0$ , the only term in the amplitude of decay of  $D^0$  is the direct amplitude  $D^0 \rightarrow f$ . At any time  $t > 0$ , there is a mixing contribution through the sequence  $D^0 \rightarrow \bar{D}^0 \rightarrow f$ . The interference of this mixing contribution with the direct decay, involves the mixing parameters:  $x$ ,  $y$ ,  $|q/p|$ ,  $\phi$ , as well as the magnitude  $r$  and strong phase  $\delta$  of the ratio of the  $\bar{D}^0 \rightarrow f$  and  $D^0 \rightarrow f$  amplitudes and plays a key role in their measurement.

The branching ratios of the Cabibbo favored (CF) decays are large and naively one might expect that one should use these decays to determine the parameters. However, if  $D^0 \rightarrow f$  is CF,  $\bar{D}^0 \rightarrow f$  is Doubly Cabibbo Suppressed (DCS) and hence the interference term is too tiny compared to leading term in the time dependent decay amplitude. For example, the decay rate for  $D^0 \rightarrow K^- \pi^+$  is,

$$\Gamma(D^0 \rightarrow K^- \pi^+) = |A_{K\pi}|^2 e^{-\Gamma t} \left[ 1 + \left| \frac{q}{p} \right| r_{K\pi} [x \sin(\delta - \phi) + y \cos(\delta - \phi)] \Gamma t + \dots \right], \quad (10)$$

where,  $A_{K\pi} \equiv A(D^0 \rightarrow K^- \pi^+)$  and the ratio of the DCS to CF amplitude is defined as:

$$-r_{K\pi} e^{-i\delta_{K\pi}} \equiv \frac{A(\bar{D}^0 \rightarrow K^- \pi^+)}{A(D^0 \rightarrow K^- \pi^+)} = \frac{A(D^0 \rightarrow K^+ \pi^-)}{A(D^0 \rightarrow K^- \pi^+)}. \quad (11)$$

Since  $r_{K\pi}$ ,  $x$  and  $y$  are much less than unity, even the linear term in  $\Gamma t$  is negligible compared to the constant term and the CF decay rate can only be used to determine  $|A_{K\pi}|$ . In a DCS mode on the other hand the constant, linear and quadratic terms in  $\Gamma t$  in the time dependent decay rate are all of the same order, allowing all three terms to be measurable. The decay rate for the DCS mode  $D^0 \rightarrow K^+ \pi^-$  is,

$$\Gamma(D^0 \rightarrow K^+ \pi^-) = |A_{K\pi}|^2 r_{K\pi}^2 e^{-\Gamma t} \left[ 1 + \left| \frac{q}{p} \right| \frac{y'_{K\pi} \cos \phi - x'_{K\pi} \sin \phi}{r_{K\pi}} \Gamma t + \left| \frac{q}{p} \right|^2 \frac{x'^2 + y'^2}{4r_{K\pi}^2} (\Gamma t)^2 \right], \quad (12)$$

where, due to the presence of a relative strong phase between the DCS and CF amplitudes, the combinations:  $x'_{K\pi} = (x \cos \delta_{K\pi} + y \sin \delta_{K\pi})$  and  $y'_{K\pi} = (y \cos \delta_{K\pi} - x \sin \delta_{K\pi})$  appear. In the linear and quadratic terms, the suppression from the small mixing parameters is compensated by the larger  $\bar{D}^0 \rightarrow f$  rate. The time dependent decay rate of  $D^0 \rightarrow K^+ \pi^-$  as well as its conjugate mode was used by BaBar and CDF collaborations to determine [2, 4]  $y'$  (from the linear term) and  $x'^2$  (from the quadratic term), assuming CP conservation.

In the case of singly Cabibbo suppressed (SCS)  $CP$  eigenstates modes, the strong phase is identically zero; and hence, the time dependent decay rate for these modes, like  $D^0 \rightarrow K^- K^+$  reduces to the simple form:

$$\Gamma(D^0 \rightarrow K^+ K^-) = |A_{KK}|^2 e^{-\Gamma t} \left[ 1 - \left| \frac{q}{p} \right| (y_{KK} \cos \phi - x_{KK} \sin \phi) \Gamma t \right].$$

Unlike the DCS modes where the term quadratic in  $\Gamma t$  is enhanced by the ratio of CF to DCS rates, in the SCS modes all time dependent terms are of the same order in  $\sin \theta_c$ , hence quadratic and higher terms in  $\Gamma t$  cannot be extracted. Assuming  $|q/p| \approx 1$  and  $\phi = 0$ , the linear term in  $\Gamma t$  can directly measure  $y$ , as has been done in Ref. [1] ( $y_{CP}$  of Ref. [1] is  $y$  for no CPV). However, the time dependent study of only the SCS  $CP$  eigenstates does not allow  $x$  to be determined, even in the limit  $|q/p| \approx 1$  and  $\phi = 0$ .

A Dalitz plot analysis [3] of  $D^0 \rightarrow K_s \pi^+ \pi^-$  has also been performed by the Belle collaboration to determine all the mixing parameters. But this has systematic errors associated with the parameterization of the resonant content

of the Dalitz plot and hence is model-dependent. Measurements of  $y_{CP}$  using  $D^0 \rightarrow K_s K^+ K^-$  as well as that of  $y'$  and  $x'^2$  using  $D^0 \rightarrow K^+ \pi^- \pi^0$  were also reported [12] at this conference.

Since the SM estimates of  $x$  and  $y$  are uncertain, the current experimental results for these parameters cannot have any clear implications for NP. However, these results already constrain the parameter space of various models. In supersymmetric models with quark-squark alignment, constraints on the up-type squark matrices have been discussed in Ref. [13]. They seem to imply squark and gluino masses above 2 TeV. A detailed analysis of various NP models has been carried out in Ref. [14]. Out of the 21 models considered, only 4 are ineffective in producing charm mixing at the observed level. For the rest of the 17 models constraints on masses and mixing parameters are obtained.

As pointed out earlier, within the SM, CPV in the  $D$  system is negligible and an observation of CPV would be a clear signal of New Physics. Babar and Belle have looked for CPV by calculating  $y'$  and  $x'^2$  for  $D^0$  and  $\bar{D}^0$  separately. They have also searched for CPV by measuring the difference of the decay rates of  $\bar{D}^0$  and  $D^0$  to SCS CP eigenstates. No evidence of CPV has been obtained. It is hence important to have a technique to accurately measure the CP violating phase.

## 5. DETERMINATION OF THE CP VIOLATING PHASE ALONG WITH OTHER MIXING PARAMETERS

A technique to accurately determine all the mixing parameters including the CP violating phase has been given in Ref. [15]. It was shown that using the DCS mode  $D \rightarrow K^{*0} \pi^0$  and its conjugate modes, one can solve for all the  $D - \bar{D}$  mixing parameters. This is possible if the  $K^{*0}/\bar{K}^{*0}$  is reconstructed both in the self tagging  $K^\pm \pi^\mp$  mode and in the CP eigenstate  $K_S \pi^0$  mode.

With the  $K^{*0}/\bar{K}^{*0}$  reconstructed in the self tagging  $K^\pm \pi^\mp$  modes, the time dependent decay rate has a form similar to that in Eq. (12). The amplitude  $|A_{K^* \pi}| \equiv |A(D^0 \rightarrow \bar{K}^{*0} \pi^0)|$  can easily be measured using the time integrated rate for the CF mode  $D^0 \rightarrow \bar{K}^{*0} \pi^0$ . The magnitude of the ratio of the DCS to CF amplitude can hence be determined using constant term in time dependent decay rate. The quadratic terms in  $\Gamma t$  in the time dependent decay rates of  $D^0 \rightarrow K^{*0} \pi^0$  and its conjugate mode  $\bar{D}^0 \rightarrow \bar{K}^{*0} \pi^0$  readily determine  $|q/p|$  and  $x'^2 + y'^2$ . The interference terms, now involve the 4 unknown parameters:  $x$ ,  $y$ ,  $\phi$  and  $\delta$ , with a known value of  $x'^2 + y'^2$ . To determine them all, an additional observable is required.

The  $K^{*0}/\bar{K}^{*0}$  in the final state could also have been reconstructed in the neutral  $K_S \pi^0$  mode, this provides the required additional observable. A unique feature of the final state  $K_S \pi^0 \pi^0$  is that it includes contributions from both  $K^{*0} \pi^0$  as well as  $\bar{K}^{*0} \pi^0$  states; the amplitude for this final state is thus a sum of the CF and DCS amplitudes,

$$|A_{K_S \pi \pi}|^2 \equiv |A(D^0 \rightarrow K_S \pi^0 \pi^0)|^2 = |A_{K^* \pi}|^2 (1 + r_{K^* \pi}^2 - 2 r_{K^* \pi} \cos \delta_{K^* \pi}). \quad (13)$$

Since the decay mode involves two neutral pions it will not be easy to perform a time dependent study. Hence, we consider only the time integrated decay rate for this mode. The amplitudes  $A(D^0 \rightarrow K_S \pi^0 \pi^0)$  and  $A(\bar{D}^0 \rightarrow K_S \pi^0 \pi^0)$  are equal since  $K_S \pi^0 \pi^0$  is a CP eigenstate. Hence, the time integrated decay rate for  $D^0 \rightarrow K_S \pi^0 \pi^0$  has the form,

$$\int_0^\infty |A(D^0(t) \rightarrow K_S \pi^0 \pi^0)|^2 dt \approx |A_{K^* \pi}|^2 \left[ 1 + \frac{q}{p} (y \cos \phi - x \sin \phi) - 2 r_{K^* \pi} \cos \delta_{K^* \pi} \right]. \quad (14)$$

Using this along with the linear terms of the time dependent decay rates of the self tagging modes allows a solution for  $\tan^2 \phi$  and for  $x/y$  with a four-fold ambiguity.  $x$  and  $y$  can thus be individually determined since  $x'^2 + y'^2$  is known. The solution obtained is finite even if  $\phi = 0$ , with a correction term of order  $(x \cos \delta_{K^* \pi} + y \sin \delta_{K^* \pi}) \sin \phi$ . Hence an accurate estimation is possible, even if  $\phi$  is tiny. It should be possible to determine  $|x|$ ,  $|y|$  to order  $7 \times 10^{-4}$ ,  $4 \times 10^{-4}$  respectively and  $\phi$  to about  $1^\circ$  at a Super-B factory with an integrated luminosity of  $50 ab^{-1}$ .

While the SCS CP eigenstates like  $D \rightarrow K^+ K^-$  cannot alone be used to determine all the mixing parameters, minimal additional information from DCS modes makes this possible. This approach may provide the optimal method to determine all the parameters with current data. For the SCS-CP eigenstates, the strong phase is identically zero

and the ratio  $r = 1$ . The coefficients of the linear term in  $\Gamma t$ , in the time dependent decay rate is a function of 3 parameters:  $x$ ,  $y$  and  $\phi$ . As pointed out earlier the quadratic and higher terms in  $\Gamma t$  cannot be extracted. However, if we also include in this analysis the quadratic terms in  $\Gamma t$  from the time dependent decay rates of DCS modes such as  $K\pi$ , all the mixing parameters can be solved without approximation. Since  $x'^2 + y'^2 = x^2 + y^2$ , these quadratic terms readily determine  $|q/p|$  and  $f^2 = x^2 + y^2$ . Alternatively,  $|q/p|$  and  $f^2$  could be measured using time integrated wrong sign relative to right sign SL decay rates. Having obtained  $|q/p|$  and  $f^2$ ,  $\phi$  and  $x/y$  can easily be determined from  $D \rightarrow K^+K^-$  and the solutions have been shown to be finite even for small  $\phi$ .

It may be further pointed out that if information from  $K^+K^-$  modes is added to that from the  $K^*\pi$  modes it helps in reducing the ambiguities in  $x$  and  $y$  from four-fold to two-fold. It has recently been proposed to use the singly Cabibbo suppressed (SCS)  $D \rightarrow K^*K$  modes to determine the mixing parameters [16, 17]. However, if  $\phi$  is zero, these methods would be feasible only if the strong phase involved is measured elsewhere.

## 6. CONCLUSIONS

Mixing in the neutral  $D$  system has been clearly established. The Standard Model predictions involve large uncertainties, obscuring detection of New Physics contributions. An observation of CP violation in  $D$  mixing would clearly imply New Physics. The  $D \rightarrow K^*\pi^0$  modes are an example where it is possible to measure the CP violating parameters as well as the mass and width differences of the two  $D$  meson mass-eigenstates using only related final states, thereby reducing systematic errors.

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